International Journal of Theoretical Physics, Vol. 32, No. 6, 1993

Finsler and Kaluza–Klein Gauge Theories

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Received April 3, 1992

A comparison of Kaluza-Klein and Finsler-type gauge theories is sketched. It is shown that the two can be related by a mapping between fiber spaces which is equivalent to a transformation from one representation of the gauge group to another. The Finsler theory lends itself to an interpretation of the mapping operators as being geometrically similar to Yang–Mills potentials. The equations of motion in this theory contain fields which are comparable to connections instead of curvatures. This gives a new geometrical framework for unified field theories.

On an m-dimensional differentiable base manifold M which has local coordinates x^μ (Greek indices $1, \ldots, m$) two different types of fiber structure $\Pi^{-1}(x)$ over a point x might be considered.

The first is a general n-dimensional fiber with coordinates z^i (Latin indices $1, \ldots, n$). This is sometimes called an "internal" space [as, for example, in Ikeda (1985, 1987)], but often simply the fiber subspace of the fiber bundle. A group G is assumed to act on the fiber and produce what physicists call gauge transformations, that is, transformations of the subspace which can be expressed locally in terms of the z coordinates. It is well known that this geometry can be used to model Kaluza-Klein theories. There are numerous examples of rigorous mathematical treatments of this type of theory, including Cho (1975) and Nash and Sen (1983).

The second type of structure is less well known, but has been around for a long time. It can be understood as arising from a special case of the first where the fiber has dimension $n = m$ of the base space and the local fiber coordinates (labeled y^μ) are taken to be related to a velocity. These are theories of Finsler type (Asanov, 1985, 1987, 1990; Miron and Anastasiei, 1987). The fiber subspace is also subject to groups of transformations. The Finsler-type theories have been generalized considerably in recent years

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902/32/6-10

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and do not necessarily correspond to traditional Finsler spaces, but still have a similar geometric approach (Kawaguchi and Miron, 1989*b*).

One purpose of this paper is to show that the Kaluza–Klein and Finsler structures are not necessarily alternative, but can exist simultaneously as complementary fiber spaces which define two different representations of the transformation group G. The projection or mapping from one fiber space to the other is accomplished by operators e_i^{μ}.

A second purpose is to outline how the Finsler approach facilitates a different way of looking at the physical interpretation of gauge geometry. For example, the connections N can correspond to fields rather than potentials as they do in Kaluza–Klein theory. The potentials can be related to the mapping operators themselves.

The discussion is begun by considering a change of the local canonical coordinates (x^{μ}, z^{i}) on the total space of the bundle. This will be distinguished below from a pure gauge transformation. The coordinate transformation is given by

$$x'^{\mu} = x'^{\mu}(x^1, \ldots, x^m), \qquad \operatorname{rank}\|X_{\nu}^{*\mu}\| = m \tag{1}$$

$$z'^{k} = M_i^{*k} z^i, \qquad \operatorname{rank}\|M_i^{*k}\| = n \tag{2}$$

where $X_{\nu}^{*\mu} = \partial x'^{\mu}/\partial x^{\nu}$.

Equation (2) distinguishes the present theory from that developed by Asanov (1985, 1987), in which z is scalar under the x transformations.

The Jacobian matrix of this coordinate transformation in the $(m+n)$-dimensional bundle space is

$$\left\| \begin{matrix} X_{\mu}^{\nu} & 0 \\ \partial M_k^i/\partial x'^{\mu}\, z'^k & M_k^i \end{matrix} \right\|$$

The natural basis of the module of the tangent vector fields in the general bundle space is $(\partial/\partial x^{\mu}, \partial/\partial z^{i})$. The $\partial/\partial x^{\mu}$ are assumed holonomic, but $[\partial/\partial z^i, \partial/\partial z^j] = f_{ij}^k(\partial/\partial z^k)$. The f's are structure constants for G in the representation defined by the z fiber. The dual basis is (dx^{μ}, dz^{i}). Then (1) and (2) correspond to the tangent space transformations

$$\frac{\partial}{\partial x'^{\mu}} = X_{\mu}^{\nu}\frac{\partial}{\partial x^{\nu}} + \frac{\partial M_k^i}{\partial x'^{\mu}} z'^k \frac{\partial}{\partial z^i}, \qquad \frac{\partial}{\partial z'^k} = M_k^i \frac{\partial}{\partial z^i} \tag{3}$$

$$dx'^{\mu} = X_{\nu}^{*\mu}\, dx^{\nu}, \qquad dz'^k = M_i^{*k}\, dz^i + \frac{\partial M_i^{*k}}{\partial x^{\nu}} z^i\, dx^{\nu} \tag{4}$$

with $X_{\alpha}^{\mu} X_{\nu}^{*\alpha} = \delta_{\nu}^{\mu}$ and $M_j^i M_k^{*j} = \delta_k^i$.

The discussion here tracks closely that of Miron and Anastasiei (1987) and others.

It is desirable to have a tangent basis which transforms covariantly under the coordinate transformation. So the basis $(\delta/\delta x^\mu, \partial/\partial z^i)$ is defined with

$$\frac{\delta}{\delta x^\mu} = \frac{\partial}{\partial x^\mu} - N^k_\mu \frac{\partial}{\partial z^k}$$

where N^k_μ is the connection, also called the nonlinear connection. The required covariance is achieved if N transforms according to

$$N'^i_\nu = M^{*i}_k N^k_\mu X^\mu_\nu + M^{*i}_k \frac{\partial M^k_j}{\partial x'^\nu} z'^j \tag{5}$$

A corresponding dual basis is $(dx^\mu, \delta z^k)$ with

$$\delta z^k = dz^k + N^k_\nu \, dx^\nu \tag{6}$$

The Finsler-type space parallels the above development except that the fiber coordinate is y^μ instead of z^i, where y^μ is a tangent vector component in a fiber space with $n = m$. In most Finsler treatments the case where

$$y^\mu = dx^\mu / ds \tag{7}$$

is taken. The increment ds is associated with an arc length of a timelike line element of the base space.

The immediate result of (7), considering (4), is that the matrix M can be identified with X so that

$$y'^\mu = X^{*\mu}_\nu y^\nu \tag{8}$$

and (3)-(5) hold with M replaced by X. See, for example, Miron and Anastasiei (1987).

Metrics $g_{\mu\nu}$ and g_{ik} can be defined on the base space and the fiber space, respectively.

The bundle line element is

$$dS^2 = g_{\mu\nu} \, dx^\mu \, dx^\nu + g_{ik} \, \delta z^i \, \delta z^k$$

This metric is block diagonal in $m+n$ dimensions and is scalar under the coordinate transformation when

$$g'_{\alpha\beta} = X^\mu_\alpha X^\nu_\beta g_{\mu\nu}, \qquad g'_{jm} = M^i_j M^k_m g_{ik}$$

The usual procedure in Kaluza-Klein theories is to take the natural basis (dx^μ, dz^i) instead of the contravariant basis $(dx^\mu, \delta z^i)$ so that the $m+n$ metric is the familiar

$$\left\| \begin{matrix} g_{\mu\nu} + N^i_\mu N^k_\nu g_{ik} & N^i_\mu g_{ik} \\ N^k_\nu g_{ik} & g_{ik} \end{matrix} \right\|$$

as is easily derived using (6). The $m \times m$ block is then the metric on which the Kaluza-Klein model is built.

Instead of using the nonlinear connection itself in an equation of motion, a new set of connection coefficients is derived which involves partial derivatives of N and the Yang-Mills form $\mathscr{F}^k_{\mu\nu} = \partial_\mu N^k_\nu - \partial_\nu N^k_\mu + f^k_{ij} N^i_\mu N^j_\nu$ for N independent of z (Cho, 1975).

The new connections are actually similar to curvatures and the equation of motion is comparable to an equation of geodesic deviation.

This is to be contrasted with some Finsler-type theories where the equation of motion is the geodesic equation for the system, which is

$$\frac{\delta y^\mu}{\delta s} = \frac{dy^\mu}{ds} + N^\mu_\nu \frac{dx^\nu}{ds} = 0 \tag{9}$$

The connection is N itself. A quantity like $\mathscr{F}^k_{\mu\nu}$ is then only a curvature.

In the usual physical interpretation of the Finsler quantities, N is identified as a potential and $\mathscr{F}$ as a field in deference to the Kaluza-Klein tradition. However, it will be seen that a different physical interpretation is possible in the Finsler case whereby a potential is part of the metric, but not geometrically similar to the nonlinear connection N. It is a field F that will be related to N and this field will appear in an appropriate place in an equation of motion like (9), which is a geodesic equation.

Usually, the Finsler and Kaluza-Klein theories have been studied separately and there has been little exploration of their relation to each other. A notable exception is the work of Ikeda (1985, 1987, 1989), who has developed some of the possibilities of a mapping

$$y^\mu = e^\mu_k z^k \tag{10}$$

Asanov (1985) has used a similar mapping with $m = n$. The mapping operator $e(x)$ is an m-bein for $m \leq n$, but not for $m > n$. The general concept of vierbein or tetrad mappings is very old. However, in nearly every case previously considered the mapping is from the fiber space to the base space instead of between fiber spaces as described here. A classic example of previous approaches is Kibble (1961).

In this type of mapping three distinct cases have to be considered, $m < n$, $m = n$, and $m > n$. In a manner somewhat similar to that of Bergmann (1983), although for a different geometrical context, the case of $m < n$ can be thought of as an embedding of the m-space in the n-space and the case $m > n$ can be thought of as a projection of the n-space onto the m-space.

It will be seen that there is a case with $m = n$ which can correspond to the Yang-Mills $SU(2)$ field and an example of the case $m > n$ which can produce a theory corresponding to $U(1)$ or electromagnetism. These are the two examples which will be examined in some detail.

For the $m < n$ case, in which the dimension of the fiber is greater than that of the base space, a complete set of basis vectors for the fiber could include m e^k_μ's, but must also include $n - m$ additional vectors. The e operators satisfy $e^\mu_k e^k_\nu = \delta^\mu_\nu$, but $e^k_\mu e^\mu_i \neq \delta^k_i$.

The fiber transformations (2) and (8) are related by

$$X^{*\mu}_\nu = e^\mu_k M^{*k}_i e^i_\nu, \qquad M^{*k}_i = \delta^k_i - e^k_\mu e^\mu_i + e^k_\mu X^{*\mu}_\nu e^\nu_i$$

so that $M^{*k}_i = \delta^k_i$ when $X^{*\mu}_\nu = \delta^\mu_\nu$.

For the $m = n$ case,

$$X^{*\mu}_\nu = e^\mu_k M^{*k}_i e^i_\nu, \qquad M^{*k}_i = e^k_\mu X^{*\mu}_\nu e^\nu_i$$

since $e^\mu_k e^k_\nu = \delta^\mu_\nu$ and $e^k_\mu e^\mu_i = \delta^k_i$.

For the case of $m > n$, $m - n$ additional basis vectors are required beyond the n e^μ_k's, since $e^\mu_k e^k_\nu \neq \delta^\mu_\nu$ and $e^k_\mu e^\mu_i = \delta^k_i$. The transformations are related by

$$X^{*\mu}_\nu = \delta^\mu_\nu - e^\mu_k e^k_\nu + e^\mu_j M^{*j}_k e^k_\nu, \qquad M^{*k}_i = e^k_\mu X^{*\mu}_\nu e^\nu_i$$

The case of $m = n$, which is the simplest, is now considered in more detail. The mapping (10) is a linear change of basis for representations of the fiber group. Under this change of basis the structure constants of the group transform as third-rank tensors (Gilmore, 1974) so the properties of the Lie algebra of the group are preserved.

Under this mapping, $\delta y^\mu = e^\mu_k \delta z^k$ and the nonlinear connection in the y-bundle is $N^\mu_\nu = e^\mu_k N^k_\nu$.

The line element of the bundle is

$$dS^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu + h_{\mu\nu}\, \delta y^\mu\, \delta y^\nu$$

This is scalar under the m-bein mapping if

$$h_{\mu\nu} = g_{ik} e^i_\mu e^k_\nu$$

The $m \times m$ block in the bundle metric for natural tangent space bases with the y fiber is $g_{\mu\nu} + h_{\alpha\beta} N^\alpha_\mu N^\beta_\nu$. So a Kaluza–Klein theory can be built from this metric of Finsler type. However, the main concern here is with covariant bases and in this case the base metric is unchanged by the m-bein mapping. The bundle metric is still block diagonal with elements $g_{\mu\nu}$ and $h_{\mu\nu}$.

While the metric $g_{\mu\nu}$ is unchanged by the m-bein mapping, it is affected in the Finsler case by the gauge transformations themselves. It will be seen that a new type of theory can be produced from these gauge-transformed metrics.

In order to demonstrate this, consider gauge transformations which are transformations of the fiber coordinates only. The transformation group is initially understood to act on the internal or fiber space and not directly on the base space coordinates. They are restricted for convenience to those which can be expressed in the form

$$\bar{z}^k = Z_i^{*k} z^i \tag{11}$$

where the matrix Z^* is $n \times n$ and nonsingular. This can be viewed as a special case of the transformations (1) and (2) with $X_\nu^{*\mu} = \delta_\nu^\mu$ and $M_i^{*k} = Z_i^{*k}$. This is sometimes called a pure gauge transformation.

Due to the m-bein mapping (10) to the y fiber the corresponding gauge transformations in the y space are

$$\bar{y}^\mu = e_k^\mu Z_i^{*k} e_\nu^i y^\nu = Y_\nu^{*\mu} y^\nu$$

which defines the nonsingular matrix Y^*. For a general vector component A^i, $\bar{A}^\alpha = e_k^\alpha Z_i^{*k} A^i = Y_\beta^{*\alpha} e_i^\beta A^i$, that is, the e mapping and the gauge transformations commute.

A key requirement of Finsler theory is that the norm or length of the y vector is preserved under the gauge transformation:

$$\bar{g}_{\mu\nu} \bar{y}^\mu \bar{y}^\nu = g_{\alpha\beta} y^\alpha y^\beta$$

which implies that the base space metric transforms as

$$\bar{g}_{\mu\nu} = g_{\alpha\beta} Y_\mu^\alpha Y_\nu^\beta, \qquad Y_\alpha^{*\mu} Y_\nu^\alpha = \delta_\nu^\mu \tag{12}$$

This transformation of the base metric is equivalent to requiring that the Finsler metric function $F^2(x, y) = g_{\alpha\beta} y^\alpha y^\beta$ be a scalar under the gauge transformation. A number of other implications of this type of gauge are discussed in Beil (1992). The main point here is that the gauge transformation induces a new form for the base space metric, but does not directly involve a transformation of the base space coordinates.

The new metric could, in general, be a velocity or y-dependent metric. In the traditional Finsler theory a metric $f_{\mu\nu} = \frac{1}{2}\partial^2 F^2/\partial y^\mu \, \partial y^\nu$ is used to compute connections and curvatures. In order to introduce explicitly the y dependence, the operator e could be dependent on both x and y, but this case will not be considered here.

The gauge transformations obviously act on the y and z fiber metrics $h_{\mu\nu}$ and g_{ik}. These metrics have been investigated by Asanov and others (Asanov, 1985, 1987; Asanov *et al.*, 1988). The Finsler condition (12) actually implies what is sometimes called a "soldering" of the fiber space onto the base space. There is a soldering form θ which, for example, would give $g_{\mu\nu} = \theta_\mu^\alpha \theta_\nu^\beta h_{\alpha\beta}$. See, among others, Ivanenko and Sardanashvily (1983) for a discussion of this. A different point of view, which involves an almost Hermitian structure relating the fiber and base metrics, is given by Kawaguchi and Miron (1989*a*,*b*). The soldering provides a means for the action of a pure gauge transformation to be imposed back on space-time.

The Finsler framework generally offers a means to couple the fiber or gauge space with the base space (space-time). Such a coupling is expected to be a constituent of a gauge theory of gravitation (Ivanenko and Sardanashvily, 1983).

It is convenient to take the form of the gauge transformation matrix in (11) to be

$$Z_i^{*k} = \delta_i^k + \omega_i^k \tag{13}$$

The $n \times n$ matrix ω_i^k contains the parameters of the gauge transformation group. For example, the number of parameters would be n^2 for $Gl(n, r)$. For interesting physical cases the number of independent parameters would be reduced, say, to three for ω_i^k as a 4×4 representation of the quaternion group $Gl(1, q) \approx SU(2, c)$. This would give a familiar Yang–Mills case.

The inverse of (13) is

$$Z_j^i = \delta_j^i + \psi_j^i$$

with ψ defined by

$$\psi_j^i + \omega_j^i + \psi_k^i \omega_j^k = 0$$

to produce $Z_i^{*k} Z_j^i = \delta_j^k$.

Under the m-bein mapping,

$$Y_\nu^{*\mu} = \delta_\nu^\mu + e_k^\mu \omega_i^k e_\nu^i, \qquad Y_\nu^\mu = \delta_\nu^\mu + e_k^\mu \psi_i^k e_\nu^i \tag{14}$$

The new metric, from (12), is

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + g_{\alpha\nu} e_j^\alpha \psi_k^j e_\mu^k + g_{\mu\beta} e_j^\beta \psi_k^j e_\nu^k + g_{\alpha\beta} e_j^\alpha \psi_k^j e_\mu^k e_i^\beta \psi_m^i e_\nu^m \tag{15}$$

A metric-like quantity e_{ji} can be defined by

$$e_{ji} = g_{\alpha\beta} e_j^\alpha e_i^\beta, \qquad e_{ji} e_\nu^i = g_{\alpha\nu} e_j^\alpha \tag{16}$$

When (16) is used in (15),

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + (2e_{jm}\psi_k^j + e_{ji}\psi_k^j \psi_m^i) e_\mu^k e_\nu^m = g_{\mu\nu} + U_{km} e_\mu^k e_\nu^m \tag{17}$$

where U_{km} has the form of a metric.

Equation (17) has a similar form to the general metric $g_{\mu\nu} + g_{km} N_\mu^k N_\nu^m$ which characterizes Kaluza–Klein theory. Here, however, the m-bein or e operator appears in place of the nonlinear connection N as representing the Yang–Mills potential. Once the metric (17) is given, the connections and curvatures can be computed according to standard procedures. The fields involve partial derivatives of the e operators and are geometrically similar to the connections N. The fields then appear in the geodesic equations.

The $m = n$ case could be developed further, but at this point it is more instructive to turn to a case with $m > n$.

For $m > n$ the mapping (10) is a projection of the n-space onto the m-space. The operators e^μ_k are no longer m-beins, since $e^\mu_k e^k_\nu \neq \delta^\mu_\nu$. They will simply be called projection operators.

With some general vector q^μ in the m-space can be associated a vector q^k in the n-space,

$$q^k = e^k_\mu q^\mu$$

However, for $m > n$ the projection does not uniquely determine q^μ since for $q^k = 0$ the equations $e^k_\mu q^\mu = 0$ will have $m - n$ linearly independent solutions for q^μ.

This implies that $m - n$ new basis vectors for the m-space must be defined in addition to the e^μ_k.

These basis vectors are taken to be q^μ_P with

$$e^k_\mu q^\mu_P = 0, \qquad q^\mu_P q^Q_\mu = \delta^Q_P$$

and the index $P = 1, \ldots, m - n$. If the direction in space-time defined by any e^μ_k or q^μ_P is spacelike, then that vector would have a pure imaginary factor. This slight complication will be omitted here.

The discussion here is formally similar to that of Rosen and Tauber (1984), although the geometric context is different. It is not difficult to show, as they have done, that

$$\delta^\mu_\nu = e^\mu_k e^k_\nu + q^\mu_P q^P_\nu$$

This means that the gauge transformation matrices can be expressed as

$$Y^{*\mu}_\alpha = e^\mu_k Z^{*k}_i e^i_\alpha + q^\mu_P q^P_\alpha \tag{18}$$

so that $Y^{*\mu}_\alpha = \delta^\mu_\alpha$ for $Z^{*k}_i = \delta^k_i$.

Similarly,

$$Y^\alpha_\nu = e^\alpha_j Z^j{}_m e^m_\nu + q^\alpha_P q^P_\nu$$

which gives $Y^{*\mu}_\alpha Y^\alpha_\nu = \delta^\mu_\nu$.

For gauge transformations like (13), the immediate result is (14) again, so that (15)-(17) have exactly the same appearance.

A simple example of this type of gauge transformation is a one-dimensional representation of the Abelian group $U(1)$. The transformation matrix (13) becomes the single element $Z^* = 1 + \omega$. This is not the compact representation which is usually considered.

For $m = 4$ the space is taken as Minkowskian, so that

$$\delta^\mu_\nu = e^\mu e_\nu + q^\mu_P q^P_\nu$$

with $P = 1, 2, 3$ and the q^μ_P all spacelike. The e^μ are 4-vectors in a timelike direction.

The transformation matrices are, from (18),

$$Y^{*\mu}_{\nu} = e^{\mu}e_{\nu} + e^{\mu}\omega e_{\nu} + q^{\mu}_{P}q^{P}_{\nu} = \delta^{\mu}_{\nu} + e^{\mu}\omega e_{\nu}$$

$$Y^{\mu}_{\nu} = \delta^{\mu}_{\nu} + e^{\mu}\psi e_{\nu}, \qquad 1+\psi = (1+\omega)^{-1} \tag{19}$$

$$Y^{*\mu}_{\nu} Y^{\nu}_{\alpha} = \delta^{\mu}_{\alpha}$$

It is assumed that the initial base space metric has the Lorentz form $\eta_{\alpha\beta}$, so that the gauge transformation is from a local inertial space to one which expresses the dynamics of the system. So (17) is

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + (2\psi + \psi^2)e_{\mu}e_{\nu}$$

The transformation can be simplified a bit by reparametrization to a parameter χ defined by $1+\omega = (1+\chi)^{-1/2}$, which gives

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + \chi e_{\mu}e_{\nu}$$

This is the same form as the metric

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + kB_{\mu}B_{\nu} \tag{20}$$

The metric class (20) has been studied extensively (Beil, 1987, 1989, 1992; Kawaguchi and Miron, 1989*a*,*b*, 1991; Miron and Radivoiovici-Tatoiu, 1989; Asanov and Kawaguchi, 1990). It has been shown to produce an equation of motion which is just the Lorentz charged particle equation, where B_{μ} is related to the electromagnetic potential by

$$B_{\mu} = A_{\mu} + \partial\Lambda/\partial x^{\mu} \tag{21}$$

The Lorentz equation comes from the geodesic equation (9), where the connection N^{μ}_{ν} is computed from the metric (20). This is possible whenever a "metric" condition is imposed such that the derivative of the metric which is covariant under the x-coordinate transformation is assumed to be zero. This result is derived in numerous discussions of Finsler space and also parallels any general relativistic derivation of the Christoffel connection. The connections thus involve partial derivatives of the B vectors (or the e vectors) and are related to the electromagnetic field $F_{\mu\nu}$. This is worked out in detail in Beil (1992). The connection N is equal to F plus a term which becomes zero in the equation of motion.

The gauge relation (21) is not the same as the gauge transformation imposed on the potential components A_{μ} by (19). The latter is given by

$$\bar{A}_{\mu} = A_{\mu} + \psi(e^{\alpha}A_{\alpha})e_{\mu}$$

If there exists a scalar function $\eta(x)$ such that $\partial\eta/\partial x^{\mu} = \psi(e^{\alpha}A_{\alpha})e_{\mu}$, then the field components $F_{\mu\nu} = \delta A_{\nu}/\delta x^{\mu} - \delta A_{\mu}/\delta x^{\nu}$ are invariant under

(19). The field form itself is invariant in any case. Since $F_{\mu\nu}$ is also invariant under (21), this implies $F_{\mu\nu} = \partial B_\nu/\partial x^\mu - \partial B_\mu/\partial x^\nu$.

There are several possible forms for η. These are related to a theory of "natural" gauges (Beil, 1991) and will be discussed in subsequent work.

A parenthetical remark should also be made to the effect that the projection vector e^μ does not have to be timelike. Gauge transformations can be defined for e spacelike or even null. In general, for the Abelian group, the matrices are the same as (19) except that $1+\psi = [1+(e^\alpha e_\alpha - 1)\omega]/(1+e^\alpha e_\alpha \omega)$. The physical significance of these transformations remains to be developed.

The differences between the Kaluza-Klein and Finsler types of theory can be summarized as follows:

In Kaluza-Klein theories the Yang-Mills potentials B are identified with the connections N, the fields become curvatures which are computed in terms of the partial derivatives of these potentials (and any structure constants), and the equations of motion are comparable to equations of geodesic deviation.

In the traditional Finsler theory N is also identified as a potential, but is sometimes used in a geodesic equation like (9) interpreted as the equation of motion. A quantity like $\mathscr{F}$ is usually taken to be a curvature.

In the present interpretation of Finsler theory the Yang-Mills potentials B are identified with the m-beins or projection operators e, and the fields F are gauge invariant, but are geometrically related to the connections N. The equations of motion are comparable to the geodesic equations. Specifically, here the operators e are not identified with the connections as in the work of Ikeda and in numerous other gauge gravitation theories. This has the advantage that the type of mathematical problem pointed out by Ivanenko and Sardanashvily (1983) related to the identification of the tetrads with the connections can be avoided. Here the gauge potentials are put on the same geometrical footing as gravitational tetrad potentials.

It should be understood that mathematically what is here called "Kaluza-Klein" and "Finsler" are two sides of the same coin. They are related through the e mapping and therefore have similar geometric properties. The difference is that the Finsler theory in the covariant basis points to a different physical interpretation, as outlined in the preceding paragraphs.

So the Finsler approach offers a new way of producing unified field theories. A novel theory of this type, in which electromagnetism as a $U(1)$ gauge is incorporated directly into the space-time metric, has been given above and in previous work (Beil, 1987, 1989, 1991, 1992). The framework for an $SU(2)$ theory was started above. Theories of $SU(3)$ and other gauges can clearly follow along the lines suggested.

ACKNOWLEDGMENTS

I am grateful to S. Ikeda and R. Miron for reading the manuscript and making valuable suggestions.

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